

Dynamic Spectrum Sharing Among Repeatedly Interacting Selfish Users With Imperfect Monitoring

Yuanzhang Xiao and Mihaela van der Schaar, *Fellow, IEEE*

Abstract—We develop a novel design framework for dynamic distributed spectrum sharing among secondary users (SUs), who adjust their power levels to compete for spectrum opportunities while satisfying the interference temperature (IT) constraints imposed by primary users. The considered interaction among the SUs is characterized by the following three unique features. First, the SUs are interacting with each other repeatedly and they can coexist in the system for a long time. Second, the SUs have limited and imperfect monitoring ability: they only observe whether the IT constraints are violated, and their observation is imperfect due to the erroneous measurements. Third, since the SUs are decentralized, they are selfish and aim to maximize their own long-term payoffs from utilizing the network rather than obeying the prescribed allocation of a centralized controller. To capture these unique features, we model the interaction of the SUs as a repeated game with imperfect monitoring. We first characterize the set of Pareto optimal operating points that can be achieved by deviation-proof spectrum sharing policies, which are policies that the selfish users find it in their interest to comply with. Next, for any given operating point in this set, we show how to construct a deviation-proof policy to achieve it. The constructed deviation-proof policy is amenable to distributed implementation, and allows users to transmit in a time-division multiple-access (TDMA) fashion. In the presence of strong multi-user interference, our policy outperforms existing spectrum sharing policies that dictate users to transmit at constant power levels simultaneously. Moreover, our policy can achieve Pareto optimality even when the SUs have limited and imperfect monitoring ability, as opposed to existing solutions based on repeated game models, which require perfect monitoring abilities. Simulation results validate our analytical results and quantify the performance gains enabled by the proposed spectrum sharing policies.

Index Terms—Repeated games, Intervention, Power control, Flow control

I. INTRODUCTION

COGNITIVE radios have increased in popularity in recent years, because they have the potential to significantly improve the spectrum efficiency. Specifically, cognitive radios enable the secondary users (SUs), who initially have no rights to use the spectrum, to share the spectrum with primary users (PUs), who are licensed to use the spectrum, as long as the PUs' quality of service (QoS), such as the throughput, is not affected by the SUs [1]. A common approach to guarantee PUs' QoS requirements is to impose *interference temperature* (IT) constraints [1][2][3][5]–[13]; that is, the SUs cannot generate an interference level higher than the interference temperature limit set by the PUs. One of the major challenges

in designing cognitive radio systems is to construct a spectrum sharing policy that achieves high spectrum efficiency while maintaining the IT constraints set by PUs.

The spectrum sharing policy, which specifies the SUs' transmit power levels, is essential to improve spectrum efficiency and protect the PUs' QoS. Since SUs can use the spectrum as long as they do not degrade the PUs' QoS, they can use the spectrum and coexist in the system for long periods of time. In general, the optimal spectrum sharing policy should allow SUs to transmit at different power levels temporally even when the environment (e.g. the number of SUs, the channel gains) remains unchanged. However, most existing spectrum sharing policies require the SUs to transmit at *constant* power levels over the time horizon in which they interact¹ [2]–[14]. These policies with constant power levels are inefficient in many spectrum sharing scenarios where the interference among the SUs is strong. Under strong multi-user interference, increasing one user's power level significantly degrades the other users' QoS. Hence, when the cross channel gains are large, the feasible QoS region is nonconvex [20]. In this case of nonconvex feasible QoS region, a spectrum sharing policy with constant power levels is inferior to a policy with *time-varying* power levels in which the users transmit in a time-division multiple-access (TDMA) fashion, because the latter can achieve the Pareto boundary of the convex hull of the nonconvex feasible QoS region.

Another important feature neglected in the design of spectrum sharing policies in recent works [2]–[11] is the selfishness of SUs, who aim to maximize their own QoS and may deviate from the prescribed spectrum sharing policy, if by doing so their QoS can be improved. Hence, the spectrum sharing policy should be *deviation-proof*, which means that selfish SUs cannot improve their QoS by deviating from the policy. In this way, selfish SUs will find it in their self-interest to follow the policy.

Given the fact that the SUs will interact with each other repeatedly when sharing the spectrum, we model the interaction among the SUs as a repeated game. In a repeated game, the stage game is played repeatedly, and a user's payoff in the repeated game is the discounted average of the stage-game payoffs (i.e. QoS in the stage games). Users can choose different actions (i.e. power levels) in different stage games, and the repeated-game payoff is a convex combination of different stage-game payoffs. A repeated-game strategy prescribes what action to take given past observations, and

Manuscript received 5 January 2012; revised 16 May 2012
Y. Xiao and M. van der Schaar are with Department of Electrical Engineering, UCLA (e-mail: {yxiao,mihaela}@ee.ucla.edu).
Digital Object Identifier 10.1109/JSAC.2012.121105.

¹Although some spectrum sharing policies go through a transient period of adjusting the power levels before the convergence to the optimal power levels, the users maintain constant power levels after the convergence.

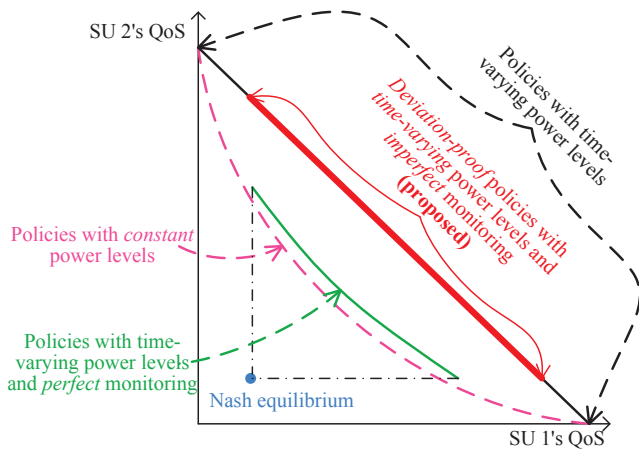


Fig. 1. An illustration of the best operating points achievable by different policies in a two-SU spectrum sharing system.

therefore, can be considered as a spectrum sharing policy. If a repeated game strategy constitutes an equilibrium, then no user can gain from deviation at any occasion. Hence, an equilibrium strategy is a deviation-proof spectrum sharing policy.

The spectrum sharing policy in a repeated game framework was studied in [15]–[18], under the assumption of *perfect* monitoring, namely the assumption that each SU can perfectly monitor the individual transmit power levels of all the other SUs. In the policies in [15]–[18], when a deviation from the prescribed policy by any user is detected, a perpetual punishment phase [15] or a punishment phase of certain duration [16][18] will be triggered. In the punishment phase, all the users transmit at the maximum power levels to create strong interference to each other, resulting in low QoS of all the users as a punishment. Due to the threat of this punishment, all the users will follow the policy in their self-interests. However, since the monitoring can never be perfect, the punishment phase, in which all the users receive low throughput, will be triggered even if no one deviates. Thus, the users' repeated-game payoffs, averaged over all the stage-game payoffs, cannot be Pareto optimal because of the low payoffs received in the punishment phases. Hence, the policies in [15]–[18] must have performance loss in practice where the monitoring is always imperfect.

Repeated games with imperfect monitoring have been studied extensively in the game theory literature. In [19], it is shown that for a general repeated game with imperfect monitoring, Pareto optimal operating points can be asymptotically achieved if certain sufficient conditions are satisfied. One sufficient condition requires the users to be able to statistically distinguish sufficiently many different actions. Translated to the spectrum sharing scenario, it requires the SUs to be able to distinguish a certain number of interference temperature levels, where the number of distinguishable IT levels grows linearly with the number of power levels each user can choose from. This requirement indicates the need for a large amount of feedback information on IT levels. Moreover, another sufficient condition requires the users to be sufficiently patient, namely they discount future payoffs arbitrarily little (i.e., their discount factors are arbitrarily close to one). This requirement

TABLE I
COMPARISON WITH RELATED WORKS IN DYNAMIC SPECTRUM SHARING.

	Power levels	Distributed	Deviation-proof	Monitoring
[2][3]	Constant	No	No	N/A
[4]–[11]	Constant	Yes	No	N/A
[12]–[14]	Constant	Yes	Yes	N/A
[15]–[18]	Time-varying	Yes	Yes	Perfect
Proposed	Time-varying	Yes	Yes	Imperfect

on the users' patience limits the scenarios to which the policy in [19] can be applied.

In this paper, we design deviation-proof spectrum sharing policies with time-varying power levels to achieve Pareto optimal operating points that are not achievable by existing policies with constant power levels [2]–[14]. We provide a systematic design approach, which first characterizes the set of Pareto optimal operating points achievable by deviation-proof policies, and then for any operating point in this set, constructs a deviation-proof policy to achieve it. The proposed policy can be easily implemented in a distributed manner. Moreover, we prove that the proposed policy can achieve Pareto optimal operating points, even when the SUs are *impatient* (namely they discount future payoffs, and their discount factor are strictly smaller than one), and have *limited* and *imperfect* monitoring ability. Specifically, their monitoring ability can be limited in that they only need to distinguish two IT levels regardless of the number of power levels each user can choose from, and their monitoring can be imperfect due to the erroneous measurements of the interference temperature.² This requirement on the users' monitoring ability is significantly relaxed compared to existing works based on repeated games, which require either perfect monitoring of all the users' individual transmit power levels [15]–[18] or sufficiently good monitoring to distinguish sufficiently many IT levels [19].

We illustrate the performance gain of the proposed policies over the existing policies in Fig. 1. We show the best operating points achievable by different classes of policies in a spectrum sharing system with two SUs. Due to the strong multi-user interference, the best operating points achievable by policies with constant power levels [2]–[14] (the dashed curve) are Pareto dominated by the best operating points achieved by policies with time-varying power levels (the straight line). The proposed policy, which are deviation-proof, can achieve a portion of the Pareto optimal operating points (the thick line). Under imperfect monitoring, the policies designed under the assumption of perfect monitoring [15]–[18] (the solid curve) have large performance loss compared to the proposed policy.

Finally, we summarize the comparison of our work with the existing works in dynamic spectrum sharing in Table I. We distinguish our work from existing works in the following categories: the power levels prescribed by the spectrum sharing policy are constant or time-varying, whether the policy can be implemented in a distributed fashion or not, whether the

²As will be described later in this paper, there is an entity that regulates the interference temperature in the system, who measures the interference temperature imperfectly and feedbacks to the users a binary signal indicating whether the constraints are violated.

policy is deviation-proof or not, and what are the requirements on the SUs' monitoring ability. The "monitoring" category is only discussed within the works based on repeated games.

The rest of the paper is organized as follows. In Section II, we describe the system model for dynamic spectrum sharing. Then, in Section III, we formulate the policy design problem using repeated games. We solve the policy design problem in Section IV. Simulation results are presented in Section V. Finally, Section VI concludes the paper.

II. SYSTEM MODEL FOR DYNAMIC SPECTRUM SHARING

We consider a system with one primary user³ and N secondary users (see Fig 2 for an illustrating example of a system with two secondary users). The set of SUs is denoted by $\mathcal{N} \triangleq \{1, 2, \dots, N\}$. Each SU has a transmitter and a receiver. The channel gain from SU i 's transmitter to SU j 's receiver is g_{ij} . Each SU i chooses a power level p_i from a finite set \mathcal{P}_i . In other words, each SU choose from discrete power levels. We assume that $0 \in \mathcal{P}_i$, namely SU i can choose not to transmit. We define SU i 's maximum transmit power as $P_i^{\max} = \max_{p_i \in \mathcal{P}_i} p_i$. The set of joint power profiles is denoted by $\mathcal{P} = \prod_{i \in \mathcal{N}} \mathcal{P}_i$, and the joint power profile of all the SUs is denoted by $\mathbf{p} = (p_1, \dots, p_N) \in \mathcal{P}$. Let \mathbf{p}_{-i} be the power profile of all the SUs other than SU i . Each SU i 's instantaneous payoff (QoS) is a function of the joint power profile, namely $u_i : \mathcal{P} \rightarrow \mathbb{R}^+$. Each SU i 's payoff $u_i(\mathbf{p})$ is decreasing in the other SUs' power levels p_j , $\forall j \neq i$. Note that we do *not* assume that $u_i(\mathbf{p})$ is increasing in p_i .⁴ But we do assume that $u_i(\mathbf{p}) = 0$ if $p_i = 0$, because a SU's payoff should be zero when it does not transmit. One example of many possible payoff functions is the SU's throughput:

$$u_i(\mathbf{p}) = \log_2 \left(1 + \frac{p_i g_{ii}}{\sum_{j \in \mathcal{N}, j \neq i} p_j g_{ji} + n_i} \right), \quad (1)$$

where n_i is the noise power at SU i 's receiver.

As in [8]–[11], there is a local spectrum server (LSS) serving as a mediating entity among the SUs. The LSS has a receiver to measure the interference temperature and a transmitter to broadcast signals, but it cannot control the actions of the autonomous SUs. The LSS could be a device deployed by the PU or simply the PU itself, if the PU manages by itself the spectrum leased to the SUs. Even when the PU is the LSS, it is beneficial to consider the LSS as a separate logical entity that performs the functionality of spectrum management. The LSS could also be a device deployed by some regulatory agency such as Federal Communications Commission (FCC), who uses it for spectrum management in that local geographic area. In both cases, the LSS aims to improve the spectrum efficiency (e.g. the sum throughput of all the SUs) and the fairness, while ensuring that the IT limit set by the PU is not violated. Note that the PU may also want to maximize the spectrum efficiency to maximize its revenue obtained from spectrum leasing, since its revenue may be proportional to the sum throughput of the SUs.

³Although we study a system with one PU as in [2]–[3][5]–[7][12], our model and design framework can be easily extended to the scenario of multiple PUs located in different geographic regions.

⁴In some scenarios with energy efficiency considerations, the payoff is defined as the ratio of throughput to transmit power, which may not monotonically increase with the transmit power.

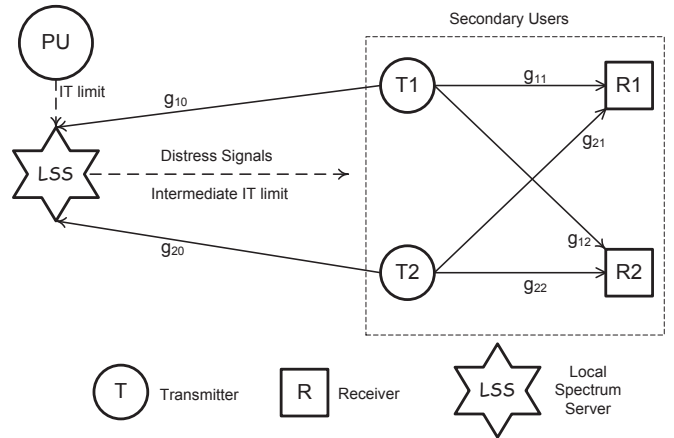


Fig. 2. An example system model with two secondary users. The solid line represents a link for data transmission, and the dashed line indicate a link for control signals. The channel gains for the corresponding data link are written in the figure. The primary user (PU) specifies the interference temperature (IT) limit to the local spectrum server (LSS). The LSS sets the intermediate IT limit to the secondary users and send distress signals if the estimated interference power exceeds the IT limit.

The LSS measures the interference temperature at its receiver imperfectly. The measurement can be written as $\sum_{i \in \mathcal{N}} p_i g_{i0} + \varepsilon$, where g_{i0} is the channel gain from SU i 's transmitter to the LSS's receiver, and ε is the additive measurement error. We assume that the measurement error has zero mean and a probability distribution function f_ε known to the LSS. We assume as in most existing works (e.g. [2]–[12]) that the IT limit \bar{I} set by the PU is known perfectly by the LSS. Although the LSS aims to keep the interference temperature below the IT limit \bar{I} , it will set a lower intermediate IT limit $I \leq \bar{I}$ to be conservative because of measurement errors. Hence, the IT constraint imposed by the LSS is

$$\sum_{i \in \mathcal{N}} p_i g_{i0} \leq I. \quad (2)$$

Even if the actual interference temperature $\sum_{i \in \mathcal{N}} p_i g_{i0}$ does not exceed the intermediate IT limit I , the erroneous measurement $\sum_{i \in \mathcal{N}} p_i g_{i0} + \varepsilon$ may still exceed the IT limit \bar{I} set by the PU. In this case, the LSS will broadcast a distress signal to all the SUs. Given the joint power profile \mathbf{p} , this false alarm probability is

$$\Gamma(\mathbf{p}) = \Pr \left(\sum_{i \in \mathcal{N}} p_i g_{i0} + \varepsilon > \bar{I} \mid \sum_{i \in \mathcal{N}} p_i g_{i0} \leq I \right), \quad (3)$$

where $\Pr(A)$ is the probability that the event A happens. We can see that a larger intermediate IT limit I enables the SUs to transmit at higher power levels, but results in a larger false alarm probability and a higher frequency of sending distress signals. Hence, there is an interesting tradeoff between the spectrum efficiency and the cost of sending distress signals.

A SU's payoff is affected by the multi-user interference $\sum_{j \in \mathcal{N}, j \neq i} p_j g_{ji}$, which is dependent on the cross channel gains among different SUs. When the multi-user interference is weak due to small cross channel gains, power control becomes less important, since one SU's power level does not affect the others' payoffs. Hence, in this paper, we focus on the more interesting scenario when the multi-user interference is strong and power control is essential for efficient interference

management. We quantify the strength of multi-user interference as follows. First, we write $\tilde{\mathbf{p}}^i = (\tilde{p}_1^i, \dots, \tilde{p}_N^i)$ as the joint power profile that maximizes SU i 's payoff subject to the IT constraint, namely

$$\tilde{\mathbf{p}}^i = \arg \max_{\mathbf{p} \in \mathcal{P}} u_i(\mathbf{p}), \text{ subject to } \sum_{i \in \mathcal{N}} p_i g_{i0} \leq I. \quad (4)$$

Since u_i is decreasing in $p_j, \forall j \neq i$, we have $\tilde{p}_j^i = 0, \forall j \neq i$. For notational simplicity, we define the maximum payoff achievable by SU i as $\bar{v}_i \triangleq u_i(\tilde{\mathbf{p}}^i)$. Then, we say a spectrum sharing scenario has strong multi-user interference if the following property is satisfied.

Definition 1 (Strong Multi-user Interference): A spectrum sharing scenario has strong multi-user interference, if the set of feasible payoffs $\mathcal{V} = \text{conv}\{\mathbf{u}(\mathbf{p}) = (u_1(\mathbf{p}), \dots, u_N(\mathbf{p})) : \mathbf{p} \in \mathcal{P}, \sum_{i \in \mathcal{N}} p_i g_{i0} \leq I\}$, where $\text{conv}(X)$ is the convex hull of X , has $N + 1$ extremal points⁵: $(0, \dots, 0) \in \mathbb{R}^N, \mathbf{u}(\tilde{\mathbf{p}}^1), \dots, \mathbf{u}(\tilde{\mathbf{p}}^N)$.

This definition characterizes the strong interference among the SUs: the increase of one SU's payoff comes at such an expense of the other SUs' payoffs that the set of feasible payoffs without time sharing is nonconvex. A spectrum sharing scenario satisfies this property when the cross channel gains among users are large [20]. In the extreme case of strong multi-user interference, simultaneous transmissions from different SUs result in packet loss, as captured in the collision model [21]. According to this definition, the set of feasible payoffs can be written as $\mathcal{V} = \text{conv}\{(0, \dots, 0), \mathbf{u}(\tilde{\mathbf{p}}^1), \dots, \mathbf{u}(\tilde{\mathbf{p}}^N)\}$. Moreover, its Pareto boundary is $\mathcal{B} = \{\mathbf{v} \in \mathcal{V} : \sum_{i=1}^N v_i / \bar{v}_i = 1, v_i \geq 0, \forall i\}$ as part of a hyperplane, which can be achieved only by SUs transmitting in a TDMA fashion.

III. FORMULATION OF THE POLICY DESIGN PROBLEM

In this section, we first formulate the interaction among the SUs as a repeated game with imperfect monitoring, and define the deviation-proof spectrum sharing policy. Then, we formally define the policy design problem and outline our design framework to solve it.

A. Formulation of The Repeated Game

Similar to [2]–[14], we assume that the system parameters, such as the number of SUs and the channel gains, remain fixed during the considered time horizon. The system is time slotted at $t = 0, 1, \dots$. We assume that the users are synchronized as in [2]–[14]. At the beginning of time slot t , each SU i chooses its power level p_i^t , and receives a payoff $u_i(\mathbf{p}^t)$. The LSS obtains the measurement $\sum_{i \in \mathcal{N}} p_i^t g_{i0} + \varepsilon^t$, where ε^t is the realization of the error ε at time slot t , and compare the measurement with the IT limit \bar{I} . The set of measurement outcomes of the comparison Y has two elements, namely $Y = \{y_0, y_1\}$. The (measurement) outcome y^t is determined by

$$y^t = \begin{cases} y_0, & \text{if } \sum_{i \in \mathcal{N}} p_i^t g_{i0} + \varepsilon^t > \bar{I} \\ y_1, & \text{otherwise} \end{cases}. \quad (5)$$

⁵The extremal points of a convex set are those that are not convex combinations of other points in the set.

We write the conditional probability distribution of the outcome y given the joint power profile \mathbf{p} as $\rho(y|\mathbf{p})$, which can be calculated as

$$\begin{aligned} \rho(y_1|\mathbf{p}) &= \int_{x \leq \bar{I} - \sum_{i \in \mathcal{N}} p_i g_{i0}} f_\varepsilon(x) dx, \\ \rho(y_0|\mathbf{p}) &= 1 - \rho(y_1|\mathbf{p}). \end{aligned} \quad (6)$$

At the end of time slot t , the LSS sends a distress signal if the outcome $y^t = y_0$. Note that the LSS does not send signals when the outcome is y_1 , and the SUs know that the outcome is y_1 by default when they do not receive the distress signal.

Note that in repeated games with perfect monitoring [15]–[18], the outcome available to each SU at time slot t is precisely the joint power profile chosen by the SUs, i.e. $y^t = \mathbf{p}^t$. We say the monitoring is imperfect if $y^t \neq \mathbf{p}^t$. In a general repeated game with imperfect monitoring, in order to achieve Pareto optimality, the set of outcomes Y should have a large cardinality, namely $|Y| \geq |\mathcal{P}_i| + |\mathcal{P}_j| - 1$ for all $i \in \mathcal{N}$ and all $j \neq i$ [19]. In contrast, our proposed policy can achieve Pareto optimality even when $|Y| = 2$ regardless of the cardinality of the SU's action set \mathcal{P}_i .

At each time slot t , each SU i determines its transmit power p_i^t based on its history, which is a collection of all the past power levels it has chosen and all the past measurement outcomes. Formally, the history of SU i up to time slot $t \geq 1$ is $h_i^t = \{p_i^0, y^0; \dots; p_i^{t-1}, y^{t-1}\} \in (\mathcal{P}_i \times Y)^t$, and that at time slot 0 is $h_i^0 = \emptyset$. The history of SU i contains private information about SU i 's power levels that is unknown to the other SUs; in contrast, we define the *public history* as $h^t = \{y^0; \dots; y^{t-1}\} \in Y^t$ for $t \geq 1$ and $h^0 = \emptyset$. The public history h^t only contains the measurement outcomes that are known to all the SUs.

In this paper, we focus on *public strategies*, in which each SU's decision depends on the public history only. Hence, each SU i 's strategy σ_i is a mapping from the set of all possible public histories to its action set, namely $\sigma_i : \sqcup_{t=0}^\infty Y^t \rightarrow \mathcal{P}_i$. Due to realization equivalence principle [25, Lemma 7.1.2], we lose nothing by only considering public strategies, in terms of the achievable Pareto optimal operating points.

The spectrum sharing policy is the joint strategy profile of all the SUs, defined as $\sigma = (\sigma_1, \dots, \sigma_N)$. The SUs are selfish and maximize their own long-term discounted payoffs. Assuming, as in [15]–[19], the same discount factor $\delta \in [0, 1)$ for all the SUs, each SU i 's (long-term discounted) payoff can be written as

$$U_i(\sigma) = (1 - \delta) \left[u_i(\mathbf{p}^0) + \sum_{t=1}^{\infty} \delta^t \cdot \sum_{y^{t-1} \in Y} \rho(y^{t-1}|\mathbf{p}^{t-1}) u_i(\mathbf{p}^t) \right],$$

where \mathbf{p}^0 is determined by $\mathbf{p}^0 = \sigma(\emptyset)$, and \mathbf{p}^t for $t \geq 1$ is determined by $\mathbf{p}^t = \sigma(h^t) = \sigma(h^{t-1}; y^{t-1})$. The discount factor represents the ‘‘patience’’ of the SUs; a larger discount factor indicates that a SU is more patient. The discount factor is determined by the delay sensitivity of the SUs' applications.

We define the deviation-proof policy as the perfect public equilibrium (PPE) of the game. The PPE prescribes a strategy profile σ from which no SU has incentive to deviate after any given history at any time slot, and thus can be considered as a deviation-proof policy. It is normally more strict than

Nash equilibrium, because it requires that the SUs have no incentive to deviate at any given history, while Nash equilibrium only guarantees this at the histories that possibly arise from the equilibrium strategy. We can also consider PPE in repeated games with imperfect monitoring as the counterpart of subgame perfect equilibrium defined in repeated games with perfect monitoring [25].

Before the definition of PPE, we introduce the concept of continuation strategy: SU i 's continuation strategy induced by any history $h^t \in Y^t$, denoted $\sigma_i|_{h^t}$, is defined by $\sigma_i|_{h^t}(h^\tau) = \sigma_i(h^t h^\tau), \forall h^\tau \in Y^\tau$, where $h^t h^\tau$ is the concatenation of the history h^t followed by the history h^τ . By convention, we denote $\sigma|_{h^t}$ and $\sigma_{-i}|_{h^t}$ the continuation strategy profile induced by h^t of all the SUs and that of all the SUs other than SU i , respectively. Then the PPE is defined as follows [25, Definition 7.1.2]

Definition 2 (Perfect Public Equilibrium): A strategy profile σ is a perfect public equilibrium if for any public history $h^t \in Y^t$, the induced continuation strategy $\sigma|_{h^t}$ is a Nash equilibrium of the continuation game, namely for all $i \in \mathcal{N}$,

$$U_i(\sigma|_{h^t}) \geq U_i(\sigma'_i|_{h^t}, \sigma_{-i}|_{h^t}), \text{ for all } \sigma'_i. \quad (7)$$

We define the equilibrium payoff as a vector of payoffs $\mathbf{v} = (U_1(\sigma), \dots, U_N(\sigma))$ achieved at the equilibrium.

B. The Policy Design Problem

The primary user or the regulatory agency aims to maximize an objective function defined on the SUs' payoffs, $W(U_1(\sigma), \dots, U_N(\sigma))$. This definition of the objective function is general enough to include the objective functions deployed in many existing works, such as [2]–[7][15][16]. An example of the objective function is the weighted sum payoff $\sum_{i=1}^N w_i U_i$, where $\{w_i\}_{i=1}^N$ are the weights satisfying $w_i \in [0, 1], \forall i$ and $\sum_{i=1}^N w_i = 1$. The PU (respectively, the regulatory agency) maximizes the objective function for the revenue (the spectrum efficiency), while maintaining the IT constraint (2). To reduce the cost of sending distress signals, a constraint on the false alarm probability is also imposed as $\Gamma(\mathbf{p}) \leq \bar{\Gamma}$, where $\bar{\Gamma}$ is the maximum false alarm probability allowed. At the maximum of the welfare function, some SUs may have extremely low payoffs. To avoid this, a minimum payoff guarantee $\gamma_i \geq 0$ is imposed for each SU i . To sum up, we can formally define the policy design problem as follows

$$\begin{aligned} \max_{\sigma} \quad & W(U_1(\sigma), \dots, U_N(\sigma)) \\ \text{s.t.} \quad & \sigma \text{ is public perfect equilibrium,} \\ & \sum_{i \in \mathcal{N}} \sigma_i(h^t) \cdot g_{i0} \leq I, \quad \forall t, \forall h^t \in Y^t, \\ & \Gamma(\sigma(h^t)) \leq \bar{\Gamma}, \quad \forall t, \forall h^t \in Y^t, \\ & U_i(\sigma) \geq \gamma_i, \quad \forall i \in \mathcal{N}. \end{aligned} \quad (8)$$

IV. SOLVING THE POLICY DESIGN PROBLEM

In this section, we solve the policy design problem (8) following the procedure outlined in Fig. 3. We first quantify the set of Pareto optimal equilibrium payoffs (i.e. the Pareto optimal payoffs that can be achieved by deviation-proof policies), then determine the optimal equilibrium payoff based on the welfare function, and finally construct the deviation-proof policy to achieve the optimal equilibrium payoff.

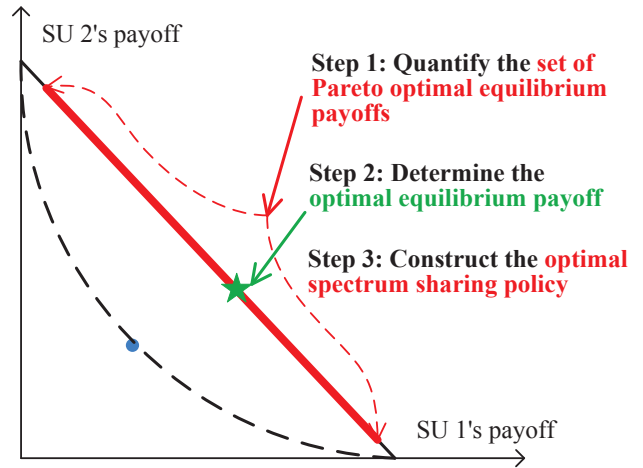


Fig. 3. The procedure of solving the design problem.

A. Quantify The Set of Pareto Optimal Equilibrium Payoffs

The first step in solving the design problem (8) is to characterize the set of Pareto optimal equilibrium payoffs for the dynamic spectrum sharing system. In particular, we are interested in the case when the SUs are impatient (their discount factor is strictly smaller than 1), as opposed to the asymptotic case when the SUs are arbitrarily patient (their discount factor goes to 1) in [15][16][19]. For repeated games with perfect monitoring, the characterization of Pareto optimal equilibrium payoffs with impatient users is provided in [18]. Our result in Theorem 1 is the first one that analytically quantifies the set of Pareto optimal equilibrium payoffs for repeated games with imperfect monitoring and impatient users.

For the spectrum sharing systems with strong multi-user interference, recall from Definition 1 that the set of feasible payoffs can be written as $\mathcal{V} = \text{conv}\{(0, \dots, 0), \mathbf{u}(\tilde{\mathbf{p}}^1), \dots, \mathbf{u}(\tilde{\mathbf{p}}^N)\}$, and that its Pareto boundary is $\mathcal{B} = \{\mathbf{v} : \sum_{i=1}^N v_i/\bar{v}_i = 1, v_i \geq 0, \forall i\}$. Now we need to determine which portion of the Pareto boundary \mathcal{B} can be achieved as equilibrium payoffs (i.e. payoffs that can be achieved by deviation-proof policies).

Before stating Theorem 1, we define the *benefit from deviation* as follows.

Definition 3 (Benefit From Deviation): We define SU j 's benefit from deviation from SU i 's payoff maximizing power profile $\tilde{\mathbf{p}}^i$ as

$$b_{ij} = \max_{p_j \in \mathcal{P}_j, \tilde{\mathbf{p}}_j^i \neq \tilde{\mathbf{p}}_j^i} \frac{\rho(y_0|\tilde{\mathbf{p}}^i) - \rho(y_0|p_j, \tilde{\mathbf{p}}_{-j}^i)}{u_j(p_j, \tilde{\mathbf{p}}_{-j}^i)/\bar{v}_j}. \quad (9)$$

Our definition of the benefit from deviation results from two intuitions. First, whether there is a benefit from deviation should depend on whether the deviation can be statistically detected. A deviation can be statistically detected only if $\rho(y_0|\tilde{\mathbf{p}}^i) < \rho(y_0|p_j, \tilde{\mathbf{p}}_{-j}^i)$. This is because $\rho(y_0|\tilde{\mathbf{p}}^i) < \rho(y_0|p_j, \tilde{\mathbf{p}}_{-j}^i)$ implies that the probability of sending the distress signal is larger when the power profile is $(p_j, \tilde{\mathbf{p}}_{-j}^i)$, in which SU j deviates from \tilde{p}_j^i to p_j , than the corresponding probability when the power profile is $\tilde{\mathbf{p}}^i$, in which SU j does not deviate. Hence, it is statistically correct for the SUs to associate the receipt of the distress signal y_0 with

the event of deviation. Since $u_j(p_j, \tilde{\mathbf{p}}_{-j}^i)/\bar{v}_j$ is always larger than 0, the benefit from deviation is negative if and only if $\rho(y_0|\tilde{\mathbf{p}}^i) < \rho(y_0|p_j, \tilde{\mathbf{p}}_{-j}^i)$. In other words, there is no benefit but only cost from deviation if the deviation can be statistically identified by the distress signal.

Second, the benefit from deviation depends on how likely deviation can be detected (reflected by $|\rho(y_0|\tilde{\mathbf{p}}^i) - \rho(y_0|p_j, \tilde{\mathbf{p}}_{-j}^i)|$), as well as how much a SU can gain from deviation (reflected by $u_j(p_j, \tilde{\mathbf{p}}_{-j}^i)/\bar{v}_j$). Since $b_{ij} < 0$, its absolute value $|b_{ij}|$ can be considered as the cost from deviation. The cost from deviation $|b_{ij}|$ increases with $|\rho(y_0|\tilde{\mathbf{p}}^i) - \rho(y_0|p_j, \tilde{\mathbf{p}}_{-j}^i)|$, the likelihood that a deviation is detected. In addition, $|b_{ij}|$ decreases with $u_j(p_j, \tilde{\mathbf{p}}_{-j}^i)/\bar{v}_j$, the payoff SU j obtains from deviation normalized by its maximum payoff.

Now we state Theorem 1, which analytically quantifies the set of Pareto optimal equilibrium payoffs.

Theorem 1: We can achieve the following set of Pareto optimal equilibrium payoffs

$$\mathcal{B}_{\underline{\mu}} = \left\{ \mathbf{v} : \sum_{i=1}^N \frac{v_i}{\bar{v}_i} = 1, \frac{v_i}{\bar{v}_i} \geq \underline{\mu}_i, \forall i \in \mathcal{N} \right\}, \quad (10)$$

where $\underline{\mu}_i \triangleq \max_{j \neq i} \frac{1 - \rho(y_0|\tilde{\mathbf{p}}^j)}{-b_{ji}}$, if and only if first, the following two sets of conditions are satisfied for all $i \in \mathcal{N}$ and for all $j \neq i$:

- Condition 1: benefit from deviation $b_{ij} < 0$;
- Condition 2: no incentive for SU i to deviate:

$$1 - \frac{u_i(p_i, \tilde{\mathbf{p}}_{-i}^i)}{\bar{v}_i} + \sum_{j \neq i} \frac{\rho(y_0|\tilde{\mathbf{p}}^i) - \rho(y_0|p_i, \tilde{\mathbf{p}}_{-i}^i)}{-b_{ij}} \geq 0, \forall p_i,$$

and second, the discount factor δ is larger than a threshold:

$$\delta \geq \underline{\delta} \triangleq \frac{1}{1 + \frac{1 - \sum_{i \in \mathcal{N}} \underline{\mu}_i}{N - 1 + \sum_{i \in \mathcal{N}} \sum_{j \neq i} (-\rho(y_0|\tilde{\mathbf{p}}^i)/b_{ij})}}. \quad (11)$$

Proof: Due to space limit, we only provide the outline of the proof here. Please refer to [27, Appendix A] for the complete proof.

The proof heavily relies on the concept of self-generating sets [26]. Simply put, a self-generating set, associated with a discount factor, is a set in which every payoff is an PPE payoff under the associated discount factor [26]. Any self-generating set is associated with a minimum discount factor; any discount factor larger than the minimum one can be associated with that self-generating set. The main contribution of the proof is to find the largest self-generating set and the associated minimum discount factor. Since we focus on the Pareto optimal equilibrium payoffs, we restrict to the self-generating sets on the Pareto boundary. This restriction allows us to obtain the analytical expression of the largest self-generating set $\mathcal{B}_{\underline{\mu}}$. Meanwhile, the sufficient and necessary conditions for $\mathcal{B}_{\underline{\mu}}$ to be self-generating are obtained. ■

Theorem 1 provides the sufficient and necessary conditions for the existence of Pareto optimal equilibrium payoffs. Condition 1 (respectively, Condition 2) ensures that at the power profile $\tilde{\mathbf{p}}^i$, SU j for any $j \neq i$ (respectively, SU i) has no incentive to deviate. When the conditions are satisfied, Theorem 1 quantifies the set of Pareto optimal equilibrium payoffs $\mathcal{B}_{\underline{\mu}}$. We can choose any payoff in $\mathcal{B}_{\underline{\mu}}$ as the deviation-proof operating point. Theorem 1 also gives us the minimum

discount factor under which any payoff in $\mathcal{B}_{\underline{\mu}}$ is achievable. We can determine the maximum level of impatience the users can have in order to achieve any payoff in $\mathcal{B}_{\underline{\mu}}$.

Remark 1: Note that we have not assumed the monotonicity of the payoff function u_i . If each SU's payoff function increases with its own transmit power, then Condition 2 in Theorem 1 holds true as long as Condition 1 is satisfied.

Remark 2: Note also that the set of Pareto optimal equilibrium payoffs $\mathcal{B}_{\underline{\mu}}$ could be empty if $\underline{\mu}_i$ is large. More precisely, $\mathcal{B}_{\underline{\mu}}$ is nonempty if and only if $\sum_{i \in \mathcal{N}} \underline{\mu}_i \leq 1$.

B. Determine The Optimal Operating Point

Since we have identified the set of Pareto optimal equilibrium payoffs $\mathcal{B}_{\underline{\mu}}$, the problem of find the optimal operating point that solves the policy design problem can be written as

$$\begin{aligned} \max_{\mathbf{v}} \quad & W(v_1, \dots, v_N) \\ \text{s.t.} \quad & (v_1/\bar{v}_1, \dots, v_N/\bar{v}_N) \in \mathcal{B}_{\underline{\mu}}, \\ & v_i \geq \gamma_i, \forall i \in \mathcal{N}. \end{aligned} \quad (12)$$

The linear constraints in the above problem can be further simplified as $v_i \geq \max\{\underline{\mu}_i \cdot \bar{v}_i, \gamma_i\}$, $\forall i \in \mathcal{N}$. Hence, we get the sufficient and necessary conditions under which the optimization problem (12) is feasible:

$$\sum_{i \in \mathcal{N}} \max\{\underline{\mu}_i, \gamma_i/\bar{v}_i\} \leq 1. \quad (13)$$

The optimization problem (12) is easy to solve when W is a convex function in (v_1, \dots, v_N) . For example, if the objective function is the weighted sum of the users' payoffs, namely $W = \sum_{i=1}^N w_i v_i$, the solution can be obtained analytically as $v_{i^*}^* = (1 - \sum_{j \neq i} \max\{\underline{\mu}_j, \gamma_j/\bar{v}_j\}) \cdot \bar{v}_i$ for $i^* = \arg \max_{j \in \mathcal{N}} w_j \bar{v}_j$, and $v_i^* = \max\{\underline{\mu}_i, \gamma_i/\bar{v}_i\} \cdot \bar{v}_i$ for all $i \neq i^*$.

C. Construct The Deviation-Proof Policy

Given the optimal payoff $\mathbf{v}^* \in \mathcal{B}_{\underline{\mu}}$, we can construct the deviation-proof policy that achieves the payoff \mathbf{v}^* . According to Definition 1, any payoff $\mathbf{v}^* \in \mathcal{B}_{\underline{\mu}}$ should be achieved by alternating among N operating points: $\mathbf{u}(\tilde{\mathbf{p}}^1), \dots, \mathbf{u}(\tilde{\mathbf{p}}^N)$. Hence, the deviation-proof policy $\sigma^*(h^t) \in \{\tilde{\mathbf{p}}^1, \dots, \tilde{\mathbf{p}}^N\}$ for any $t \geq 0$ and for any public history $h^t \in Y^t$. Since only one SU transmits in a time slot, the deviation-proof policy can also be regarded as a scheduling in a TDMA fashion. By judiciously deciding which SU can transmit in each time slot, each SU i receives a discounted expected average payoff v_i^* and has no incentive to deviate from the policy. The deviation-proof policy can be implemented by each SU in a distributed manner. The algorithm run by SU i is described in the algorithm in Table II.

The intuition of why the algorithm in Table II works is as follows. At each time slot t , each SU i calculates the indices for all the SUs, $\alpha_i(t)$, $\forall i \in \mathcal{N}$, where

$$\alpha_j(t) = \frac{v_j(t)/\bar{v}_j - \underline{\mu}_j}{1 - v_j(t)/\bar{v}_j + \sum_{k \neq j} (-\rho(y_0|\tilde{\mathbf{p}}^k)/b_{jk})}, \quad \forall j \in \mathcal{N}.$$

The index $\alpha_i(t)$ measures SU i 's "urgency" to transmit at time slot t . The SU i^* with the largest index $\alpha_{i^*}(t) = \max_i \alpha_i(t)$

TABLE II
THE ALGORITHM RUN BY USER i .

Input: The normalized target payoffs $\{v_i^*/\bar{v}_i\}_{i \in \mathcal{N}}$ given by the LSS
Initialization: Set $t = 0$, $v_j'(0) = v_j^*/\bar{v}_j$ for all $j \in \mathcal{N}$.
repeat
Calculates the index $\alpha_j(t) = \frac{v_j'(t) - \underline{\mu}_j}{1 - v_j'(t) + \sum_{k \neq j} (-\rho(y_0 \bar{\mathbf{p}}^j)/b_{jk})}$, $\forall j$
Finds the largest index $i^* \triangleq \arg \max_{j \in \mathcal{N}} \alpha_j(t)$
if $i = i^*$ then
Transmits at the power level \bar{p}_i^i
end if
Updates $v_j'(t+1)$ for all $j \in \mathcal{N}$ as follows:
if No Distress Signal Received At Time Slot t ($y^t = y_1$) then
$v_{i^*}'(t+1) = \frac{1}{\delta} \cdot v_{i^*}'(t) - (\frac{1}{\delta} - 1) \cdot (1 + \sum_{j \neq i^*} \frac{\rho(y_0 \bar{\mathbf{p}}^{i^*})}{-b_{i^*j}})$
$v_j'(t+1) = \frac{1}{\delta} \cdot v_j'(t) + (\frac{1}{\delta} - 1) \cdot \frac{\rho(y_0 \bar{\mathbf{p}}^{i^*})}{-b_{i^*j}}$, $\forall j \in \mathcal{N}, j \neq i^*$
else
$v_{i^*}'(t+1) = \frac{1}{\delta} \cdot v_{i^*}'(t) - (\frac{1}{\delta} - 1) \cdot (1 - \sum_{j \neq i^*} \frac{\rho(y_1 \bar{\mathbf{p}}^{i^*})}{-b_{i^*j}})$
$v_j'(t+1) = \frac{1}{\delta} \cdot v_j'(t) - (\frac{1}{\delta} - 1) \cdot \frac{\rho(y_1 \bar{\mathbf{p}}^{i^*})}{-b_{i^*j}}$, $\forall j \in \mathcal{N}, j \neq i^*$
end if
$t \leftarrow t + 1$
until \emptyset

TABLE III
THE INFORMATION EXCHANGE PHASE.

Events	Information obtained
SUs choose $\{\bar{\mathbf{p}}^i\}_{i \in \mathcal{N}}$	LSS: $\{\rho(y_0 \bar{\mathbf{p}}^i)\}_{i \in \mathcal{N}}$
SUs choose $(p_j^i, \bar{\mathbf{p}}_{-j}^i)$, $\forall j, p_j$	LSS: $\rho(y_0 p_j^i, \bar{\mathbf{p}}_{-j}^i)$, $\forall j, p_j$
LSS broadcasts	SU i : $\rho(y_0 \bar{\mathbf{p}}^i)$, $\rho(y_0 p_j^i, \bar{\mathbf{p}}_{-j}^i)$
SUs broadcast	LSS, SUs: b_{ij} , $\forall i, j \neq i$
SUs send to LSS	LSS: $\{\bar{v}_i\}_{i \in \mathcal{N}}$

will transmit at time slot t . When no distress signal is received (which indicates no deviation), SU i^* 's index in the next time slot is very likely to be small, in order to give the other SUs larger opportunities to transmit. However, when the distress signal is received (which indicates deviation), they calculate the indices in a different way, such that SU i^* still has a large index in the next time slot. Hence, a SU may not have the incentive to deviate, because it will lead to a smaller opportunity to transmit in the future.

Theorem 2 ensures that if all the SUs run the algorithm in Table II locally, they will achieve the optimal operating point \mathbf{v}^* , and will have no incentive to deviate.

Theorem 2: For any target payoff $\mathbf{v}^* \in \mathcal{B}_{\underline{\mu}}$, and any discount factor $\delta \geq \underline{\delta}$, the strategy generated by each user running the algorithm in Table II is PPE and achieves \mathbf{v}^* .

Proof: Due to space limit, we only provide the outline of the proof here. Please refer to [27, Appendix B] for the complete proof.

The key to the proof is to demonstrate that all the payoffs $\{v_i'(t) \cdot \bar{v}_i\}_{i \in \mathcal{N}}$, $\forall t \geq 0$ generated in the algorithm in Table II are in the self-generating set (the set of Pareto optimal equilibrium payoffs) $\mathcal{B}_{\underline{\mu}}$. ■

D. Implementation Issues

We discuss the implementation issues of our proposed design framework, which can be implemented in three phases

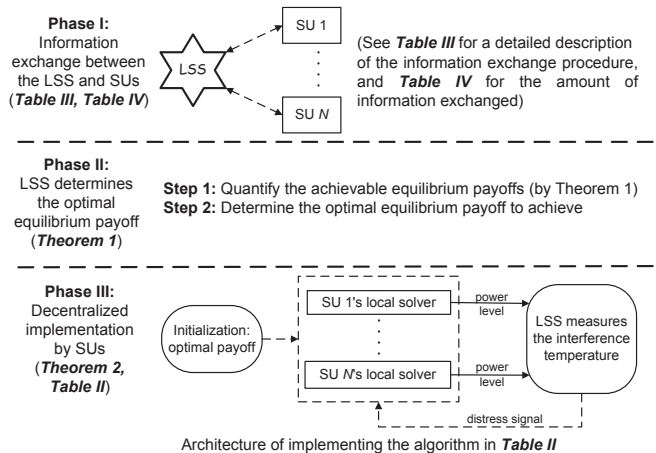


Fig. 4. Illustration of the implementation.

TABLE IV
COMPARISON OF THE TOTAL AMOUNT OF INFORMATION EXCHANGED.

	The total amount of information exchanged
[4]–[7][12]	$O(N)$ per iteration \cdot # of iterations
[10][13]	$O(N^2)$ per iteration \cdot # of iterations
Proposed	$\sum_i \sum_{j \neq i} A_j + N^2 + 1$

as illustrated in Fig. 4. In Phase I, the LSS exchanges some information with the SUs following the procedure described in Table III. In Phase II, using the information obtained in Phase I, the LSS quantifies the set of Pareto optimal equilibrium payoffs, and solves the policy design problem for the optimal equilibrium payoff. Finally in Phase III, the LSS sends the optimal equilibrium payoff to the SUs, as an input to each SU's decentralized algorithm of constructing the optimal deviation-proof policy.

1) *Overhead of information exchange:* We briefly comment on the overhead of the information exchange in the proposed framework. First, the information exchange in Phase I is necessary for the LSS to determine and for the SUs to achieve the optimal equilibrium payoff. A similar information exchange phase is proposed in [15][16][22]–[24]. The information exchange phase can be considered as a substitute for the convergence process needed by the algorithms in [4]–[7][10][12][13]. In the proposed policy, since the players implement the policy without any information exchange in Phase III, the only information exchange happens in Phase I and at the end of Phase II (when the MU broadcasts the optimal equilibrium payoff). The information exchange method in our framework is advantageous in that its duration and the amount of information to exchange are predetermined. On the other hand, the amount of information to exchange in [4]–[7][10][12][13] is proportional to the convergence time of their algorithms, which are generally unbounded. We summarize the overhead of information exchange (measured by the number of real numbers or pilot signals transmitted) in the related works in Table IV.

2) *Computational complexity:* As we can see from Table II, the computational complexity of each SU in constructing the optimal policy is very small. At each period t , each SU only needs to compute N indices $\{\alpha_j(t)\}_{j \in \mathcal{N}}$, and N normalized

payoffs $\{v'_j(t)\}_{j \in \mathcal{N}}$, all of which can be calculated by analytical expressions. In addition, although the original definition of the strategy requires each SU to memorize the entire history of measurement outcomes, in the actual implementation, each SU only needs to know the current measurement outcome and memorize N normalized payoffs $\{v'_j(t)\}_{j \in \mathcal{N}}$.

V. SIMULATION RESULTS

In this section, we demonstrate the performance gain of our spectrum sharing policy over existing policies, and validate our theoretical analysis through numerical results. Throughout this section, we use the following system parameters by default unless we change some of them explicitly. The noise powers at all the SUs' receivers are normalized as 0 dB. The maximum transmit powers of all the SUs are 10 dB, $\forall i$. For simplicity, we assume that the direct channel gains have the same distribution $g_{ii} \sim \mathcal{CN}(0, 1)$, $\forall i$, and the cross channel gains have the same distribution $g_{ij} \sim \mathcal{CN}(0, \beta)$, $\forall i \neq j$, where β is defined as the *cross interference level*. The channel gain from each SU to the LSS also satisfies $g_{i0} \sim \mathcal{CN}(0, 1)$, $\forall i$. The IT limit set by the PU is $\bar{I} = 10$ dB. The measurement error ε is Gaussian distributed with zeros mean and variance 0.1. The maximum false alarm probability is $\bar{\Gamma} = 10\%$. The SUs' payoffs are their throughput as in (1). The welfare function is the average payoff, i.e. $W = \sum_{i=1}^N \frac{1}{N} U_i$. The minimum payoff guarantee is 10% of the maximum achievable payoff, i.e. $\gamma_i = 0.1 \cdot \bar{v}_i$, $\forall i$.

A. Performance Evaluation

1) Comparison with policies with constant power levels:

We first compare the performance of the proposed policy with that of the optimal policy with constant power levels. The optimal policy with constant power levels (or "the optimal stationary policy") is the solution to the modified version of the design problem (8). First, we add an additional constraint that the power profile is constant, namely $\sigma(h^t) = \mathbf{p}^*$ for all $t \geq 0$ and for all $h^t \in Y^t$. Second, we drop the incentive constraint that σ is PPE from (8). Hence, the performance of the optimal stationary policy is the best that can be achieved by existing stationary policies [4]–[11], and is an upper bound for the deviation-proof stationary policies [12]–[14].

In Fig. 5, we compare the performance of the proposed policy and that of the optimal stationary policy under different cross interference levels and different numbers of SUs. As expected, the proposed policy outperforms the optimal stationary policy in medium to high cross interference levels (approximately when $\beta \geq 1$). In the cases of high cross interference levels ($\beta \geq 2$) and many users ($N = 5$), the stationary policy fails to meet the minimum payoff guarantees due to strong interference (indicated by zero average throughput in the figure). On the other hand, the desirable feature of the proposed policy is that the average throughput does not decrease with the increase of the cross interference level, because SUs transmit in a TDMA fashion. For the same reason, the average throughput does not change with the number of SUs.

Note that the proposed policy is infeasible (zero average throughput) when the cross interference level is very small. This is because it cannot be deviation-proof in this scenario.

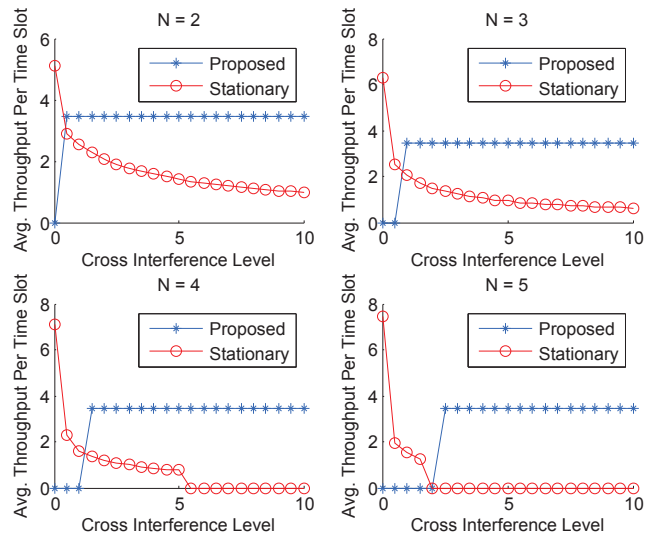


Fig. 5. Performance comparison of the proposed policy and the optimal policy with constant power levels ('stationary' in the legend) under different numbers of users and different cross interference levels. A zero average throughput indicates that there exists no feasible policy that satisfies all the constraints in the policy design problem.

When the interference level is very small, SU j can deviate from $\tilde{\mathbf{p}}^i$ and receives a high reward $u_j(p_j, \tilde{\mathbf{p}}^i_{-j})$ because the interference from SU i , $\tilde{p}_i^i g_{ij}$, is small. Hence, the benefit of deviation b_{ij} is large, and the deviation is inevitable. This observation leads to an efficient way for the LSS to check the cross interference level without knowing the channel gains. If the proposed policy is infeasible, the LSS knows that the cross interference level is low, and can switch to stationary policies.

2) Comparison with "punish-forgive" policies proposed under perfect monitoring: We also compare the proposed policy with existing policies designed under the assumption of perfect monitoring [15]–[18]. Specifically, we consider the "punish-forgive" policy in [15]–[18], which requires SUs to switch to the punishment phase of L time slots once a deviation is detected. In the punishment phase, all the SUs transmit at the maximum power levels to create high interference to the deviator⁶. A special case of the punish-forgive policy when the punishment length $L = \infty$ [15] is the celebrated "grim-trigger" strategy in game theory literature [25]. As discussed before, the punish-forgive policy works well if the SUs can perfectly monitor the individual power levels of all the SUs, because in this case, the punishment serves as a threat and will never be carried out in the equilibrium. However, when the SUs have imperfect monitoring ability, the punishment will be carried out with some positive probability, which decreases all the SUs' average payoffs.

Fig. 6 shows that the proposed policy outperforms the punish-forgive policies under different variances of measurement errors and different false alarm probabilities. For each combination of the error variance and the false alarm probability, we choose the punish-forgive policy with the optimal

⁶Note that all the SUs transmitting at the maximum power levels. For the punish-forgive policy [15]–[18], we allow the violation of the IT constraint in the punishment phase. Note that the IT constraint is never violated in the proposed policy.

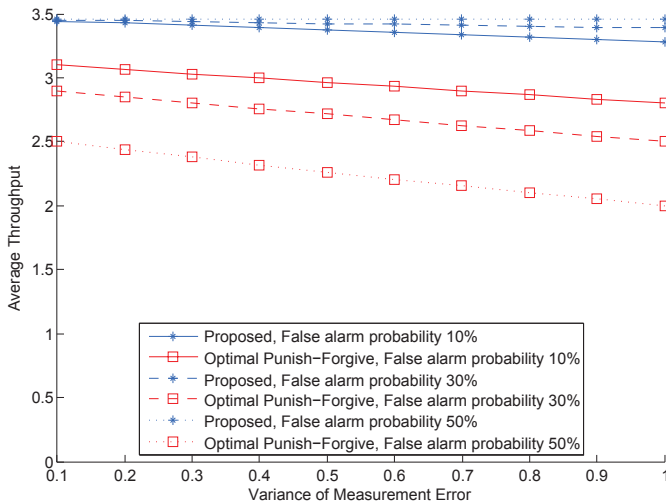


Fig. 6. Performance comparison of the proposed policy and the punish-forgive policy with the optimal punishment length under different error variances and different false alarm probabilities.

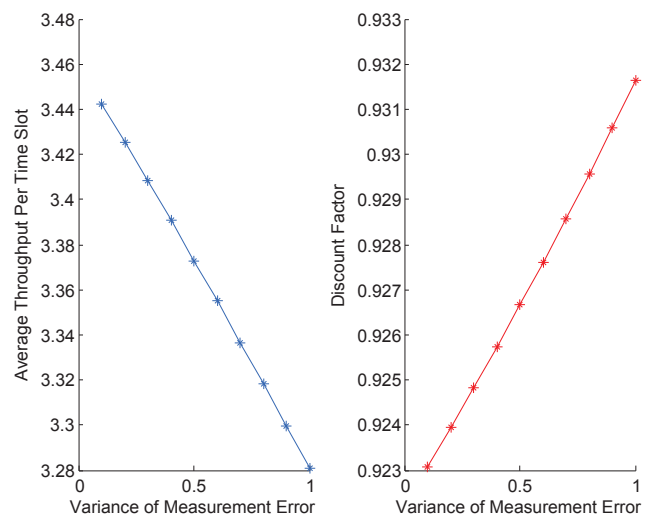


Fig. 7. The impact of the variance of the measurement error on the performance of the proposed policy and the minimum discount factor required under which the proposed policy is deviation-proof.

punishment length. The performance of punish-forgive policies degrades with the increase of the error variance and the false alarm probability, because of the increasing probability of mistakenly triggered punishments. Some interesting observation on how the performance of the proposed policy changes with the error variance and the false alarm probability is explained in details in the following subsections.

B. Impacts of Variances of Measurement Errors

Fig. 7 shows that with the increase of the variance of measurement errors, the average throughput decreases, and the SUs' patience (the discount factor) required to achieve Pareto optimal equilibrium payoffs increases. First, when the error variance increases, the intermediate IT limit I must decrease to maintain the constraint on the false alarm probability. The decrease of I leads to the decrease of SUs' maximum transmit power levels allowed, which results in the decrease of the average throughput. Another impact of the increase in the error variance is that $\rho(y_0|p_j, \tilde{\mathbf{P}}_{-j}^i) = \int_{x > \bar{I} - p_j g_{j0} - \tilde{p}_{j0}^i} f_\varepsilon(x) dx$ increases, which leads to the increase of benefit of deviation b_{ij} . Hence, the minimum discount factor $\underline{\delta}$ increases according to Theorem 1.

C. Impacts of Constraints on The False Alarm Probability

Fig. 8 shows that with the increase of the false alarm probability limit $\bar{\Gamma}$, both the average throughput and the users' patience (the discount factor) required to achieve Pareto optimal equilibrium payoffs increase. First, with an increased false alarm probability limit, the intermediate IT limit I can increase, which leads to an increase of the SUs' maximum transmit power levels and thus an increase of the users' throughput. Meanwhile, since

$$\rho(y_0|\tilde{\mathbf{P}}^i) - \rho(y_0|p_j, \tilde{\mathbf{P}}_{-j}^i) = - \int_{\bar{I} - I - g_{j0} p_j}^{\bar{I} - I} f_\varepsilon(x) dx$$

increases when I increases, the benefit of deviation b_{ij} increases. This leads to an increase of the minimum discount factor.

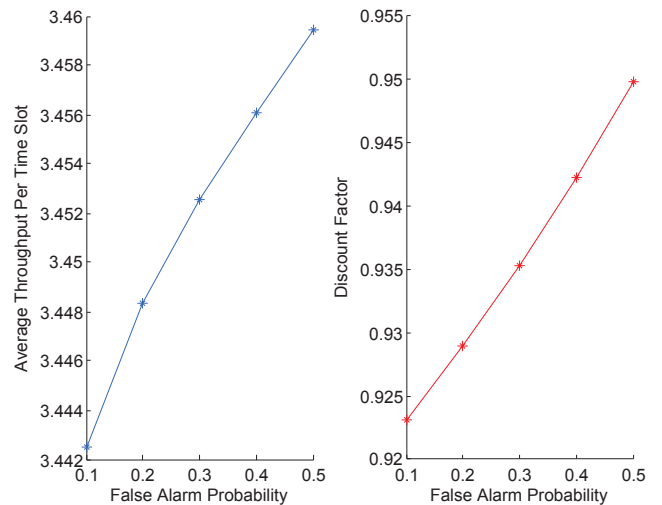


Fig. 8. The impact of the false alarm probability on the performance of the proposed policy and the minimum discount factor required under which the proposed policy is deviation-proof.

This observation indicates an interesting design tradeoff. On one hand, a smaller false alarm probability can reduce the overhead of sending distress signals, and can also relax the requirement on SUs' patience. On the other hand, a larger false alarm probability can increase the average throughput, such that the spectrum efficiency or the revenue can increase. Our theoretical results characterize such a tradeoff, which can be used to choose the optimal intermediate IT limit I .

VI. CONCLUSION

In this paper, we studied power control in dynamic spectrum sharing among SUs under the interference temperature constraint, and proposed a dynamic spectrum sharing policy that allows SUs to transmit in a TDMA fashion. The proposed policy can achieve Pareto optimal operating points that are not achievable under existing spectrum sharing policies with constant power levels. The proposed policy is amenable to

distributed implementation and is deviation-proof, in that the SUs are in their self-interests (i.e. maximizing their own QoS) to follow the policy. The proposed policy can achieve Pareto optimality even when the SUs have limited and imperfect monitoring ability: they only observe distress signals that erroneously indicate the violation of the interference temperature constraint. Simulation results validate our analytical results on the policy design and demonstrate the performance gains enabled by the proposed policy.

REFERENCES

- [1] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201-220, Feb. 2005.
- [2] X. Kang, R. Zhang, Y.-C. Liang, and H. K. Garg, "Optimal power allocation strategies for fading cognitive radio channels with primary user outage constraint," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 2, pp. 374-383, Feb. 2011.
- [3] C. W. Tan and S. H. Low, "Spectrum management in multiuser cognitive wireless networks: Optimality and algorithm," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 2, pp. 421-430, Feb. 2011.
- [4] J. Huang, R. A. Berry, and M. L. Honig, "Distributed interference compensation for wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 5, pp. 1074-1084, May 2006.
- [5] Y. Xing, C. N. Mathur, M. A. Haleem, R. Chandramouli, and K. P. Subbalakshmi, "Dynamic spectrum access with QoS and interference temperature constraints," *IEEE Trans. Mobile Comput.*, vol. 6, no. 4, pp. 423-433, Apr. 2007.
- [6] L. B. Le and E. Hossain, "Resource allocation for spectrum underlay in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5306-5315, Dec. 2008.
- [7] N. Gatsis, A. G. Marques, G. B. Giannakis, "Power control for cooperative dynamic spectrum access networks with diverse QoS constraints," *IEEE Trans. Commun.*, vol. 58, no. 3, pp. 933-944, Mar. 2010.
- [8] O. Ileri, D. Samardzija, and N. Mandayam, "Demand responsive pricing and competitive spectrum allocation via a spectrum server," in *Proc. IEEE DySPAN*, Baltimore, MD, Nov. 2005, pp. 194C202.
- [9] J. Acharya and R. Yates, "A framework for dynamic spectrum sharing between cognitive radios," in *Proc. IEEE ICC*, Glasgow, Scotland, Jun. 2007, pp. 5166C5171.
- [10] S. Sharma and D. Teneketzis, "An externalities-based decentralized optimal power allocation algorithm for wireless networks," *IEEE/ACM Trans. Netw.*, vol. 17, no. 6, pp. 1819-1831, Dec. 2009.
- [11] S. Sorooshyari, C. W. Tan, M. Chiang, "Power control for cognitive radio networks: Axioms, algorithms, and analysis," To appear in *IEEE/ACM Trans. Netw.*, 2011.
- [12] J. Huang, R. A. Berry, and M. L. Honig, "Auction-based spectrum sharing," *Mobile Networks and Applications*, vol. 11, pp. 405-418, 2006.
- [13] S. Sharma and D. Teneketzis, "A game-theoretic approach to decentralized optimal power allocation for cellular networks," *Telecommunication Systems*, pp. 1-16, 2010.
- [14] Y. Xiao, J. Park, and M. van der Schaar, "Intervention in power control games with selfish users," *IEEE J. Sel. Topics Signal Process., Special issue on Game Theory in Signal Processing*, vol. 6, no. 2, pp. 165-179, Apr. 2012.
- [15] R. Etkin, A. Parekh, and D. Tse, "Spectrum sharing for unlicensed bands," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 3, pp. 517-528, Apr. 2007.
- [16] Y. Wu, B. Wang, K. J. R. Liu, and T. C. Clancy, "Repeated open spectrum sharing game with cheat-proof strategies," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 1922-1933, 2009.
- [17] M. Le Treust and S. Lasaulce, "A repeated game formulation of energy-efficient decentralized power control," *IEEE Trans. on Wireless Commun.*, vol. 9, no. 9, pp. 2860-2869, september 2010.
- [18] Y. Xiao, J. Park, and M. van der Schaar, "Repeated games with intervention: Theory and applications in communications," To appear in *IEEE Trans. Commun.*. Available: "<http://arxiv.org/abs/1111.2456>".
- [19] D. Fudenberg, D. K. Levine, and E. Maskin, "The folk theorem with imperfect public information," *Econometrica*, vol. 62, no. 5, pp. 997-1039, Sep. 1994.
- [20] S. Stańczak and H. Boche, "On the convexity of feasible QoS regions," *IEEE Trans. Inf. Theory*, vol. 53, no. 2, Feb. 2007.
- [21] J. Park and M. van der Schaar, "Cognitive MAC protocols using memory for distributed spectrum sharing under limited spectrum sensing," *IEEE Trans. Commun.*, vol. 59, no. 9, pp. 2627-2637, Sep. 2011.
- [22] C. Cordeiro and K. Challapali, "C-MAC: A cognitive MAC protocol for multichannel wireless networks," in *Proc. Symposium on Dynamic Spectrum Access Networks (DySPAN07)*, pp. 147-157, Apr. 2007.
- [23] M. Timmers, S. Pollin, A. Dejonghe, L. van der Perre, and F. Catthoor, "A distributed multichannel MAC protocol for multihop cognitive radio networks," *IEEE Trans. Veh. Technol.*, vol. 59, no. 1, pp. 446-459, Jan. 2010.
- [24] A. De Domenico, E. C. Strinati, and M. G. Di Benedetto, "A survey on MAC strategies for cognitive radio networks," *IEEE Commun. Surveys Tutorials*, vol. 14, no. 1, pp. 21-44, 2012.
- [25] G. J. Mailath and L. Samuelson, *Repeated Games and Reputations: Long-run Relationships*. Oxford, U.K.: Oxford Univ. Press, 2006.
- [26] D. Abreu, D. Pearce, and E. Stacchetti, "Toward a theory of discounted repeated games with imperfect monitoring," *Econometrica*, vol. 58, no. 5, pp. 1041-1063, 1990.
- [27] Y. Xiao and M. van der Schaar, "Dynamic Spectrum Sharing Among Repeatedly Interacting Selfish Users With Imperfect Monitoring," Available at: <http://arxiv.org/abs/1201.3328>.

Yuanzhang Xiao received the B.E. and M.E. degree in electrical engineering from Tsinghua University, Beijing, China, in 2006 and 2009, respectively. He is currently pursuing the Ph.D. degree in the Electrical Engineering Department at University of California, Los Angeles. His research interests include game theory, optimization, communication networks, and network economics.

Mihaela van der Schaar is Chancellor's Professor of Electrical Engineering at University of California, Los Angeles. She is an IEEE Fellow, a Distinguished Lecturer of the Communications Society for 2011-2012 and the Editor in Chief of IEEE Transactions on Multimedia. She received an NSF CAREER Award (2004), the Best Paper Award from IEEE Transactions on Circuits and Systems for Video Technology (2005), the Okawa Foundation Award (2006), the IBM Faculty Award (2005, 2007, 2008), the Most Cited Paper Award from EURASIP: Image Communications Journal (2006), the Gamenets Conference Best Paper Award (2011) and the 2011 IEEE Circuits and Systems Society Darlington Award Best Paper Award. She holds 33 granted US patents. For more information about her research visit: <http://medianetlab.ee.ucla.edu/>